Correction/measurement of rooms in the low frequency range

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EXACT#2

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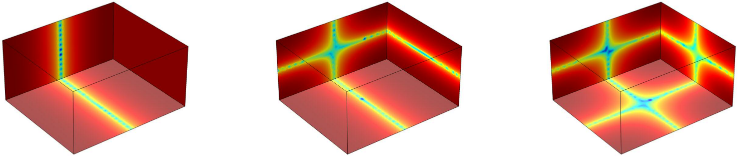
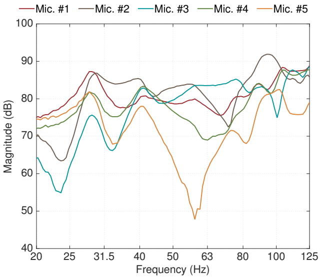
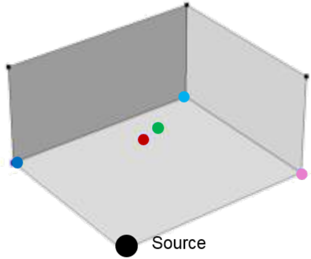
Listening Rooms at Low Frequencies (1)







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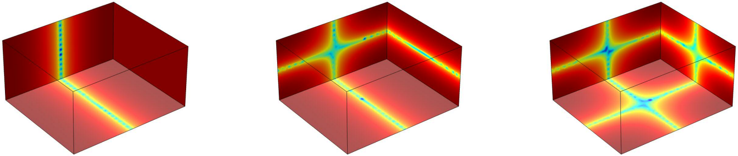
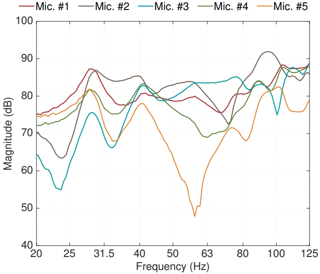




Helmholtz equation

$$\Delta p + k^2 p = 0,$$
$$k = \frac{\omega}{c}$$

Boundary conditions

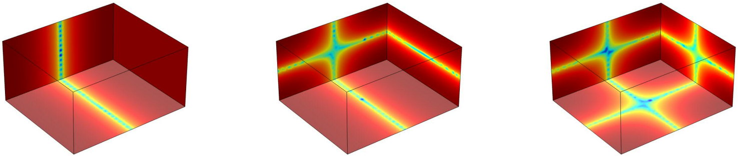
$$\vec{\nabla} p \cdot \vec{n} = -j\frac{\rho\omega}{Z_n} p$$

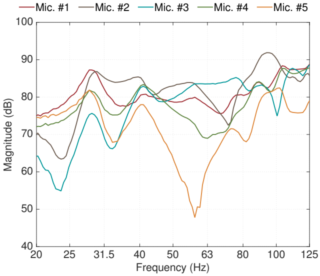
Pressure field in the frequency domain

$$p(x,y,z,\omega) = j\rho\omega c^2 q_0 \sum_{n=1}^{\infty} \frac{\Psi_n(x,y,z)\Psi_n(x_0,y_0,z_0)}{K_n(\omega^2 - \omega_n^2 + 2j\delta_n\omega)}$$

with $K_n = \int_z \int_y \int_x \Psi_n(x,y,z)dxdydz$

Listening Rooms at Low Frequencies (1)





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$$\Delta p + k^2 p = 0,$$
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Boundary conditions

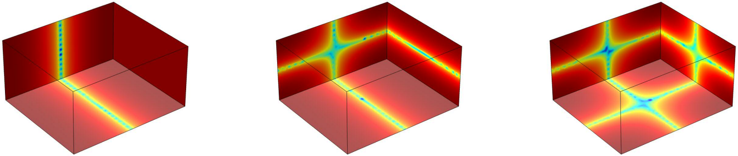
$$\vec{\nabla} p \cdot \vec{n} = -j \frac{\rho\omega}{Z_n} p$$

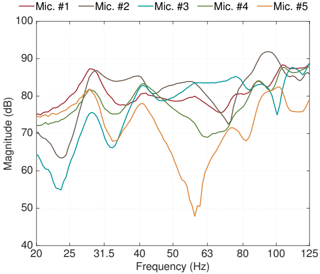
In rectangular room and ideally rigid walls

$$\Psi_n(x,y,z) = \cos\left(\frac{m_x \pi x}{l_x}\right) \cos\left(\frac{m_y \pi y}{l_y}\right) \cos\left(\frac{m_z \pi z}{l_z}\right)$$

and $f_n = \frac{c}{2}\sqrt{\left(\frac{m_x}{l_x}\right)^2 + \left(\frac{m_y}{l_y}\right)^2 + \left(\frac{m_z}{l_z}\right)^2}$

Listening Rooms at Low Frequencies (1)





Helmholtz equation

$$\Delta p + k^2 p = 0,$$
$$k = \frac{\omega}{c}$$

Boundary conditions

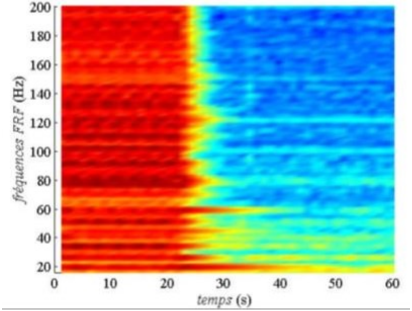
$$\vec{\nabla} p \cdot \vec{n} = -j \frac{\rho \omega}{Z_n} p$$

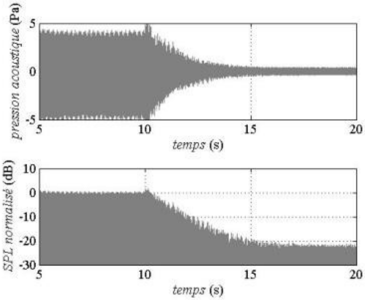
In other cases

Eigenfunctions $\Psi_n(x, y, z)$ not accessible by measurement (only numerical model).

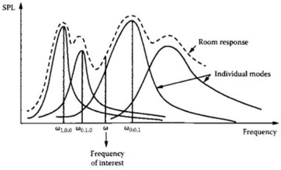
Listening Rooms at Low Frequencies (2)

What happens in the time domain?



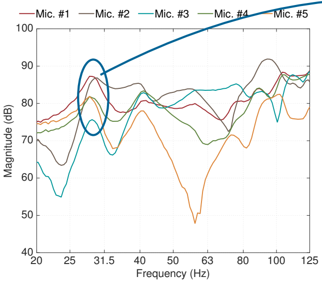


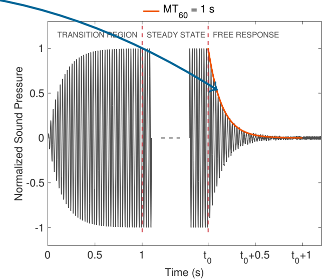
Listening Rooms at Low Frequencies (2)



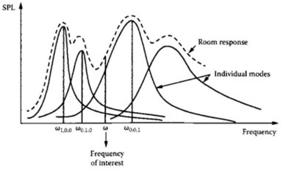
Quality factor of mode n

$$H_n(\omega) = \frac{j\omega}{\omega^2 + 2j\delta_n\omega - \omega_n^2}$$
$$\rightarrow Q_n = \frac{\omega_n}{2\delta_n} = \frac{\omega_n\tau_n}{2}$$



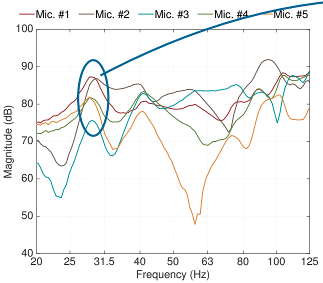


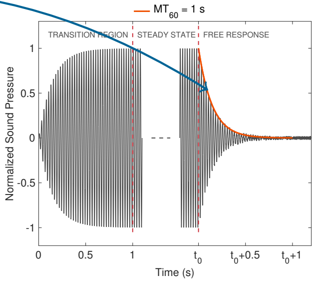
Listening Rooms at Low Frequencies (2)



Modal decay time of mode n:= decay of 60 dB

$$MT60_n = \frac{3\,ln(10)\,Q_n}{\pi\,f_n}$$



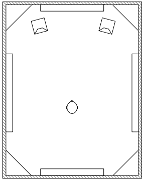


→ 1. How to improve the listening experience in any room at low frequencies?

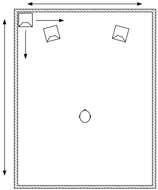
→ 2. How to measure the room acoustic characteristics at low frequencies?

Correction Strategies

- Passive means



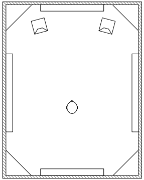
Passive absorbers



Optimal room design / configuration

Correction Strategies

- Passive means



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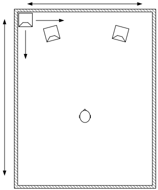
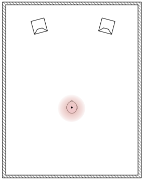
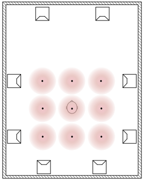


Optimal room design / configuration

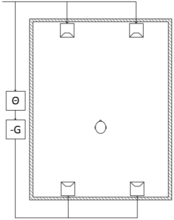
- Active equalisation



Single input



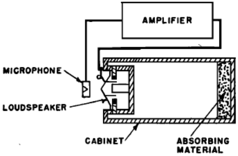
Multiple inputs



Plane wave propagation

Correction Strategies

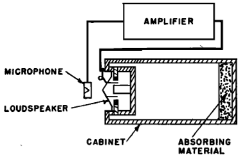
- Active absorption



Olson and May, *J. Acoust. Am.*, 1953.

Correction Strategies

- Active absorption



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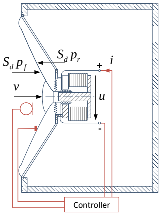
$\rightarrow$ Electroacoustic absorbers

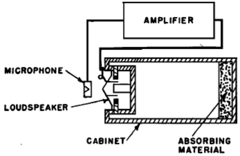
Direct



Nicholson, *PhD thesis*, 1994

Correction Strategies

- Active absorption



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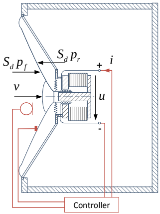
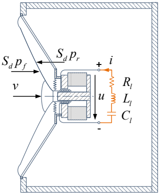


Nicholson, *PhD thesis*, 1994

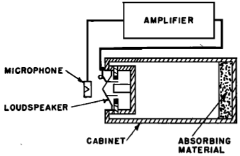
Shunt-based



Boulandet, *PhD thesis*, 2012

Correction Strategies

- Active absorption



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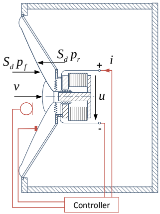

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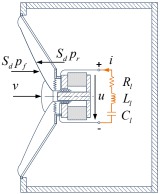
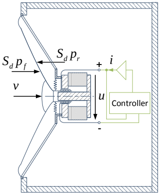


Boulandet, *PhD thesis*, 2012

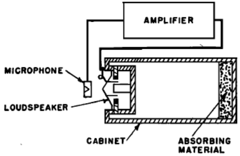
Self-sensing



Boulandet et al., *Acta Acustica united with Acustica*, 2016

Correction Strategies

- Active absorption



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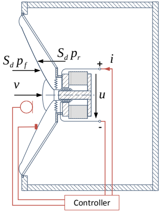

Nicholson, *PhD thesis*, 1994

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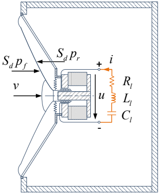

Boulandet, *PhD thesis*, 2012

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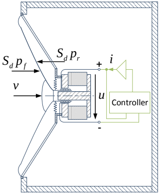
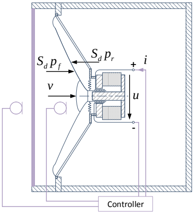


Boulandet et al., *Acta Acustica united with Acustica*, 2016

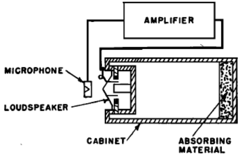
Hybrid passive/active



Galland et al., *Applied Acoustics*, 2005

Correction Strategies

- Active absorption



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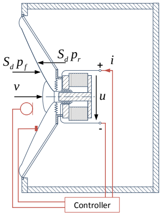

Nicholson, *PhD thesis*, 1994

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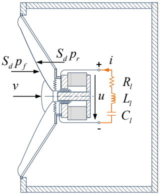
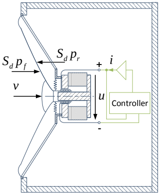


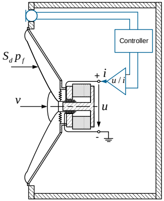
Boulandet, *PhD thesis*, 2012

Self-sensing



Boulandet et al., *Acta Acustica united with Acustica*, 2016

Hybrid sensor-/shunt-based



Rivet et al., *IEEE CST*, 2017

Overview of the presentation

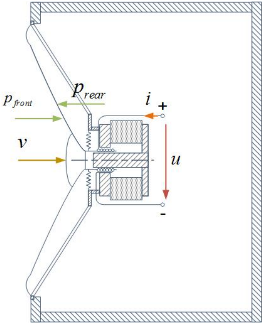
- **New concepts allow efficient sound absorption in the low-frequency range**
  → Presentation of the Electroacoustic Absorber (EA) concept
- **General performance can be measured with standard facilities (1D)**
  → Example of experimental assessment of EA (for design purpose)
  → Highlight why these standard measurement are not transposable to real room applications
- **Some 3D performance can be derived from simulations/in situ measurements**
  → Present optimization procedures based on time decay with room simulations
  → Present some experimental results (time decay) measured in real rooms
- **Still, limited tools to measure their performance in situ**
  → Present preliminary results (space distribution) obtained through COMSOL simulations

1. Electroacoustic absorbers design
   - Electroacoustic absorbers: general principle
   - Electroacoustic absorbers: hybrid sensor-/shunt-based control
   - Performance assessment in 1D Case

2. From 1D to 3D
   - Identifying target Impedance in 3D
   - Optimisation of Multiple Degree-of-Freedom Target Impedance

3. In situ performance Evaluation
   - Frequency responses
   - Modal decay times
   - Spatial distribution of pressure

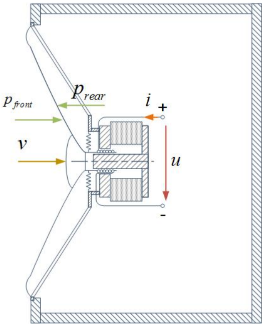
General description



Mobile equipment: M_{ms}, R_{ms}, C_{ms}
Moving coil: R_e, L_e
Electrodynamic transduction: $B\ell$
Enclosure: V_b

Diaphram area: S_d

General description



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Moving coil: R_e, L_e
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Enclosure: V_b

Diaphram area: S_d

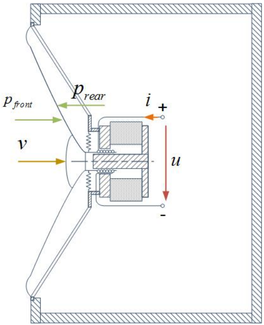
Newton's law on the mobile equipment

$$M_{ms}\frac{dv(t)}{dt} = S_d(p_{front}(t) - p_{rear}(t)) - B\ell i(t) - R_{ms}v(t) - \frac{1}{C_{ms}}\int v(t)dt$$

Mesh law on the electric circuit

$$u(t) = \left(L_e\frac{di(t)}{dt} + R_e i(t)\right) - B\ell v(t)$$

General description



Mobile equipment: M_{ms}, R_{ms}, C_{ms}
Moving coil: R_e, L_e
Electrodynamic transduction: $B\ell$
Enclosure: V_b

Diaphram area: S_d

Newton's law on the mobile equipment (Laplace domain: $s = j\omega$)

$$Z_{ms}(s)V(s) = S_d P_{front}(s) - B\ell I(s)$$

where $Z_{ms}(s) = sM_{ms} + R_{ms} + \dfrac{1}{sC_{mc}}$

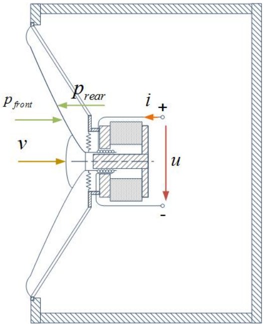
and $C_{mc} = \dfrac{C_{ms}V_b}{V_b + \rho c^2 S_d^2 C_{ms}}$

Mesh law on the electric circuit (Laplace domain: $s = j\omega$)

$$U(s) = Z_e(s)I(s) - B\ell V(s)$$

where $Z_e(s) = sL_e + R_e$

General description



Mobile equipment: M_{ms}, R_{ms}, C_{ms}
Moving coil: R_e, L_e
Electrodynamic transduction: $B\ell$
Enclosure: V_b

Diaphram area: S_d

Newton's law on the mobile equipment (Laplace domain: $s = j\omega$)

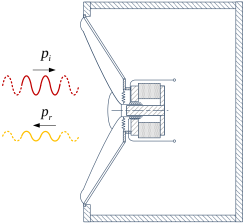
$$Z_{ms}(s)V(s) = S_d P_{front}(s) - B\ell I(s)$$

If $I(s)$ is controlled:

We can write the apparent acoustic impedance of the diaphragm:

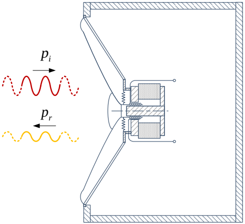
$$Z_s(s) = \frac{P_{front}(s)}{V(s)}$$

Absorption performance (under normal incidence - 1D)



- Incident pressure p_i
- Reflected pressure
  $$p_r = \Gamma(s) p_i = \frac{Z_s(s) - Z_c}{Z_s(s) + Z_c} p_i$$
  (under normal incidence)
- Total pressure $p_{front} = p_i + p_r$

Absorption performance (under normal incidence - 1D)



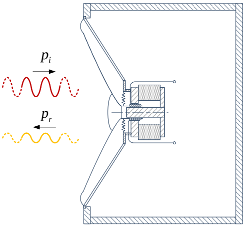
Sound absorption coefficient

$$\alpha(f) = \frac{I_a}{I_i} = 1 - \left| \frac{Z_s(f) - Z_c}{Z_s(f) + Z_c} \right|^2$$

Bandwidth of efficient sound absorption *BW*

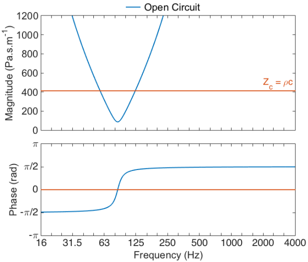
Frequency range over which $\frac{1}{2}\mathcal{I}_{tot} \leq \mathcal{I}_{ideal\ case}$
$\rightarrow \alpha_{th} \simeq 0.83$

Absorption performance (under normal incidence - 1D)

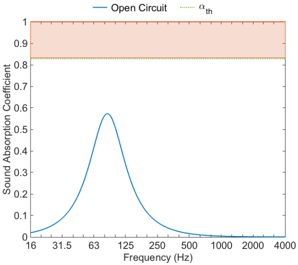


Example: open-circuit loudspeaker ($I = 0$)

$$Z_s(s) = \frac{Z_{ms}(s)}{S_d} = \frac{1}{S_d}\left[sM_{ms} + R_{ms} + \frac{1}{sC_{mc}}\right]$$



Specific acoustic impedance



Sound absorption coefficient

Hybrid Sensor-/Shunt-Based Control

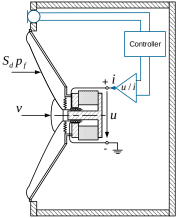
Constitutive laws

$$Z_{ms}(s)V(s) = S_d P_{front}(s) - B\ell I(s)$$

$$I(s) = \Theta(s) P_{front}(s)$$

Acoustic impedance

$$Z_s(s) = \frac{Z_{ms}(s)}{S_d - B\ell\Theta(s)}$$



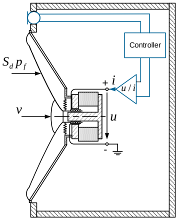
Hybrid Sensor-/Shunt-Based Control

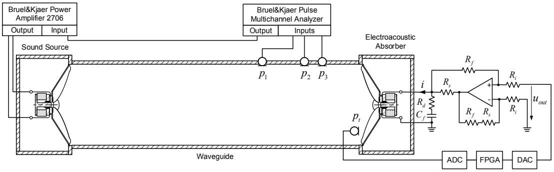
Transfer function

$$\Theta(s) = \frac{I(s)}{P_{front}(s)} = \frac{S_d Z_{st}(s) - Z_m(s)}{B\ell\, Z_{st}(s)}$$

Target impedance

$$Z_{st}(s) = s\frac{\mu M_{ms}}{S_d} + R_{st} + \frac{\mu}{s S_d C_{mc}}$$





ISO 10534-2

$$\hat{\Gamma}_{12} = \frac{H_{12} - H_I}{H_R - H_{12}}.e^{j2k.x1}$$

$$\begin{cases} H_I = e^{-jk(x_1 - x_2)} \\ H_R = e^{jk(x_1 - x_2)} \end{cases}$$

Hybrid Sensor-/Shunt-Based Control

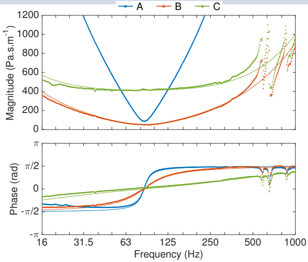
Transfer function

$$\Theta(s) = \frac{I(s)}{P_{front}(s)} = \frac{S_d Z_{st}(s) - Z_m(s)}{B\ell\, Z_{st}(s)}$$

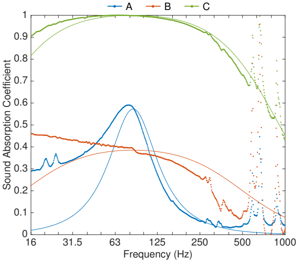
Target impedance

$$Z_{st}(s) = s\frac{\mu M_{ms}}{S_d} + R_{st} + \frac{\mu}{s S_d C_{mc}}$$

Case	μ	R_{st}
A	1	R_{ms}/S_d
B	0.15	$\rho c/8$
C	0.15	ρc



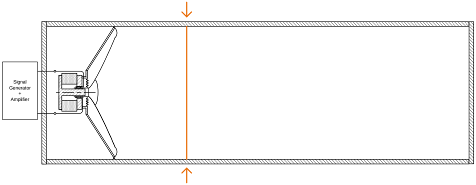
Sound absorption coefficient

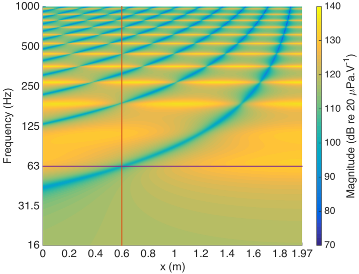


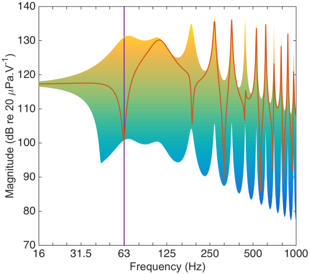
Specific acoustic impedance

Rivet et al., *IEEE Transactions on Control Systems Technology*, 2017

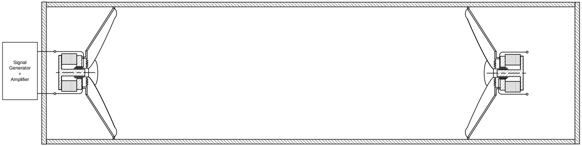
With Hard Wall

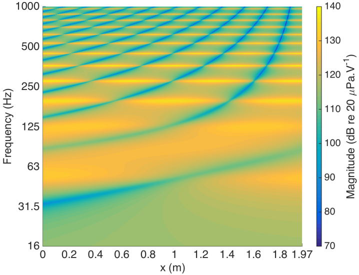


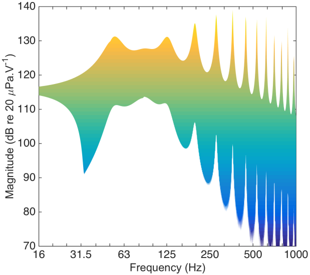




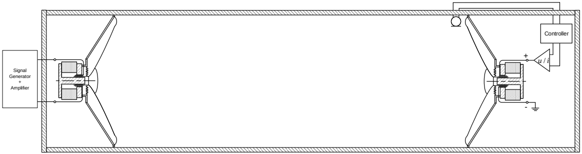
With a Loudspeaker

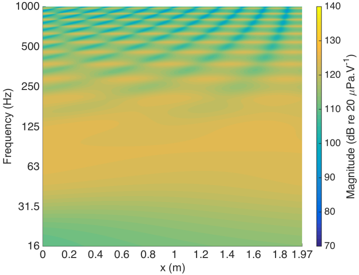


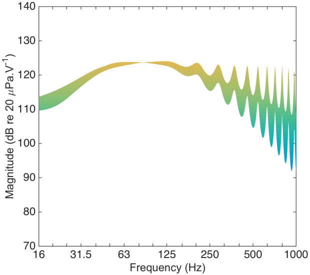




With an Electroacoustic Absorber





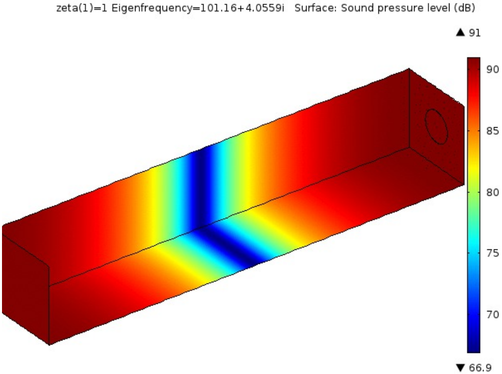


1 Electroacoustic absorbers design
- Electroacoustic absorbers: general principle
- Electroacoustic absorbers: hybrid sensor-/shunt-based control
- Performance assessment in 1D Case

2 From 1D to 3D
- Identifying target Impedance in 3D
- Optimisation of Multiple Degree-of-Freedom Target Impedance

3 In situ performance Evaluation
- Frequency responses
- Modal decay times
- Spatial distribution of pressure

1D Case under Normal Incidence

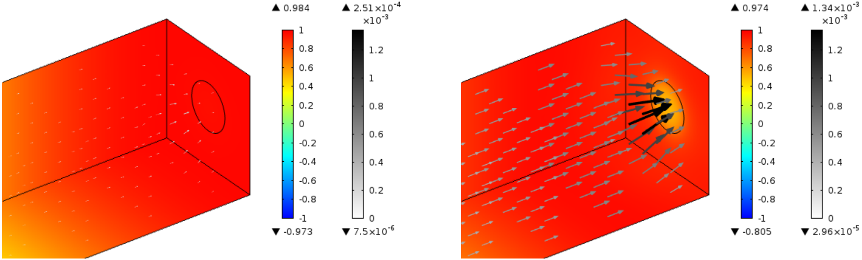


Duct dimensions: $L = 1.7$ m, $W = H = 30$ cm
Target acoustic resistance: $R_s = \zeta \rho c$

Modal decay time of mode n

$$MT_{60_n} = \frac{3\,ln(10)}{\delta_n}$$

1D Case under Normal Incidence

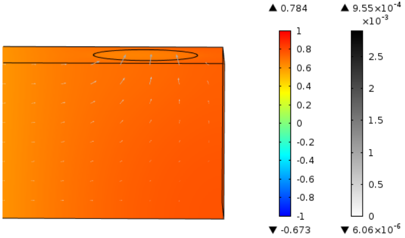


$R_s = \rho c$ $\qquad$ $R_s = 0.15\rho c$

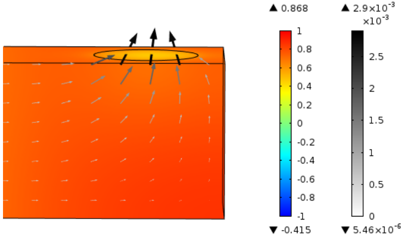
$r_{abs} = 6\,\text{cm}$, $S_{abs} = 113\,\text{cm}^2$ (12.6 % of the wall area)

Sound pressure (color), active sound intensity (gray)

1D Case under Grazing Incidence



$R_s = \rho c$



$R_s = 0.32\rho c$

$r_{abs} = 10\,\text{cm},\ S_{abs} = 314\,\text{cm}^2$

Sound pressure (color), active sound intensity (gray)

In rooms?

- Design of demonstration prototypes

→ Value of target impedance?

Optimization on Modal Decay Time

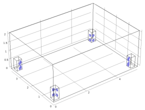
As seen before, the Modal Decay Times only depend on eigenvalues $\omega_n + j\delta_n$ (invariant with position)

$\rightarrow$ can we derive optimal values of acoustic impedance by minimizing MT_{60n} for a given set of room resonance frequencies?

Optimization with COMSOL: output δ_n for a given set of resonance frequencies

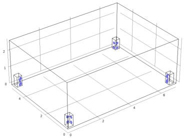
Small room S

$L = 5.33\text{m}, W = 3.76\text{m}, H = 2.13\text{m}$



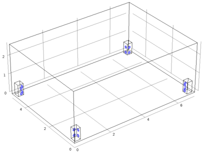
Medium room M1

$L = 7.02\text{m}, W = 5.10\text{m}, H = 2.70\text{m}$



Medium room M2

$L = 7.87\text{m}, W = 6.36\text{m}, H = 3.48\text{m}$



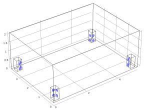
Modal decay time of mode n

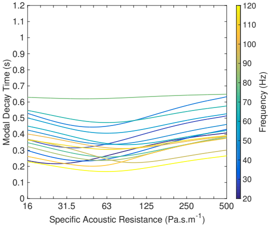
$$MT_{60_n} = \frac{3\, ln(10)}{\delta_n}$$

Modal Decay Time

Small room S

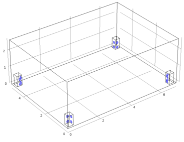
$L = 5.33\text{m}, W = 3.76\text{m}, H = 2.13\text{m}$

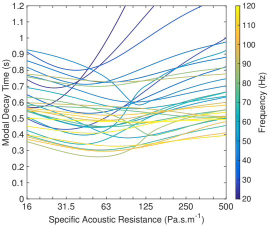




Medium room M1

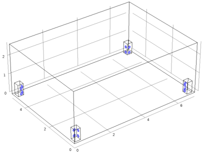
$L = 7.02\text{m}, W = 5.10\text{m}, H = 2.70$ m

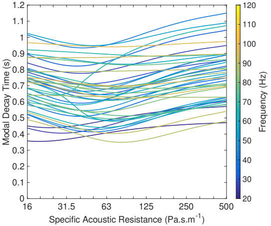




Medium room M2

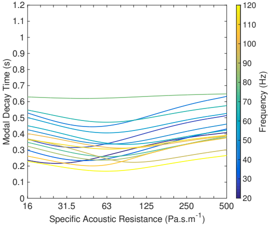
$L = 7.87\text{m}, W = 6.36\text{m}, H = 3.48\text{m}$



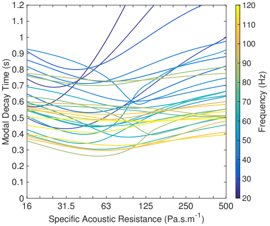


Modal Decay Time

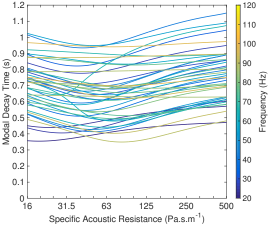
Small room S



Medium room M1

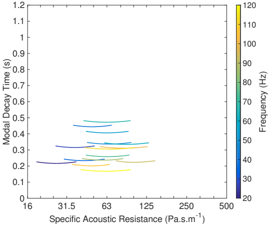


Medium room M2

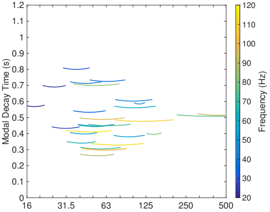


Modal Decay Time

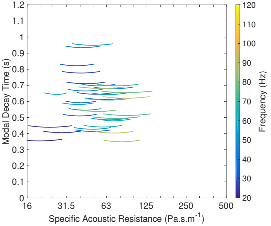
Small room S

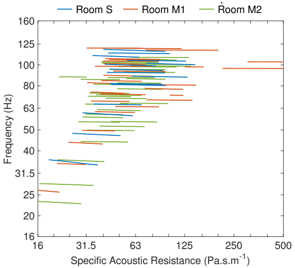


Medium room M1

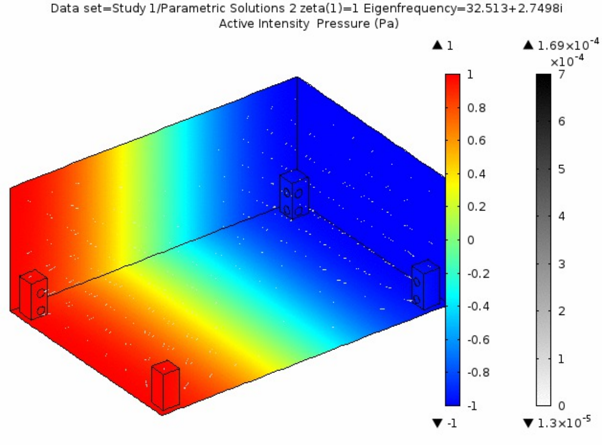


Medium room M2





Active Sound Intensity



Mode (1,0,0) with $R_{s_{abs}} = \zeta\rho c$ Sound pressure (color), active sound intensity (gray)

Design

Method to prescribe frequency-dependent resistances on the EA

Multiple degree-of-freedom target impedance

$$Z_{st_{n-DOF}}(\omega) = \frac{1}{\sum_{k=1}^{n} \frac{1}{Z_{st_k}(\omega, \nu_{2k-1}, R_{st_{2k-1}}, \nu_{2k})}}$$

where

$$Z_{st_k}(\omega, \nu_{2k-1}, R_{st}, \nu_{2k}) = j\omega \frac{M_{ms}}{S_d \nu_{2k-1}} + R_{st} + \frac{1}{j\omega\, S_d \nu_{2k}\, C_{mc}}$$

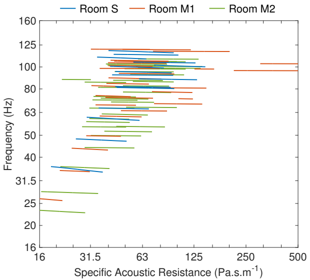
with conditions

$$\nu_1 = \nu_{M_1} - \sum_{k=2}^{n} \nu_{2k-1} \quad \text{and} \quad \nu_{2n} = \nu_{C_1} - \sum_{k=1}^{n-1} \nu_{2k}$$

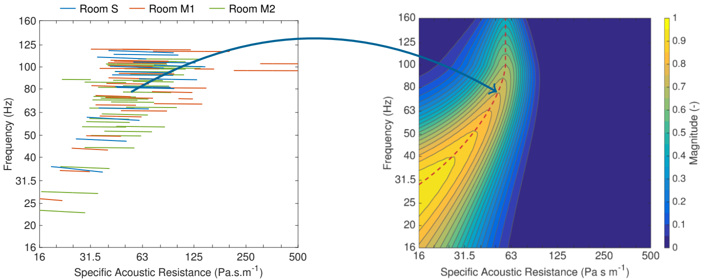
Modified sound absorption coefficient

$$\tilde{\alpha}(R_s, f) = 1 - \left| \frac{Z_s(f) - R_s}{Z_s(f) + R_s} \right|^2$$

Performance Optimisation



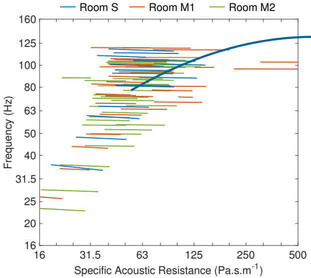
Performance Optimisation

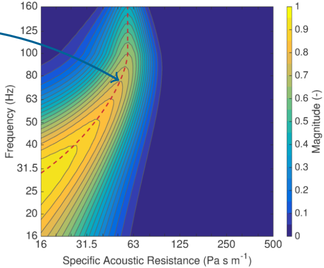


Weighting function of optimal resistances

$$R_{s_{opt}}(R_s, f) = (a_1 f + b_1)\exp\left(-\frac{\left(R_s - g\, R_{s_p}(f)\right)^2}{2(a_2 f + b_2)}\right)$$

Performance Optimisation





Weighting function of optimal resistances

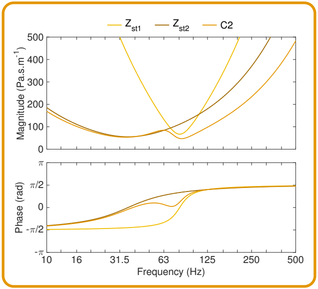
$$R_{s_{opt}}(R_s, f) = (a_1 f + b_1)\exp\left(-\frac{\left(R_s - g\, R_{s_p}(f)\right)^2}{2(a_2 f + b_2)}\right)$$

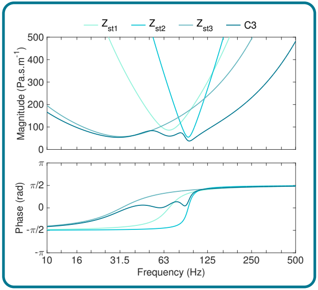
Optimisation strategy

$$\mathcal{A}_2 = \int\int \max\left(\tilde{\alpha}(R_s, f) - \alpha_{th}, 0\right) R_{s_{opt}}(R_s, f)\,\mathrm{d}R_s\,\mathrm{d}f$$

Performance analysis

$Z_{st_{2-DOF}} = Z_{st_1} // Z_{st_2}$ (C2)

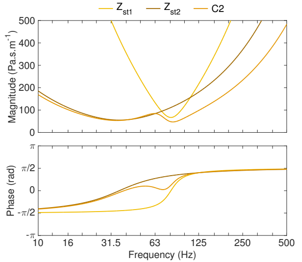


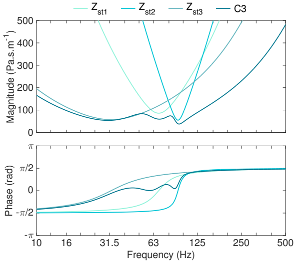


$Z_{st_{3-DOF}} = Z_{st_1} // Z_{st_2} // Z_{st_3}$ (C3)

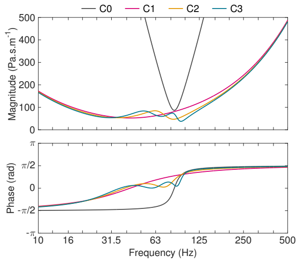
Performance analysis

$Z_{st_{2-DOF}} = Z_{st_1} // Z_{st_2}$ (C2)



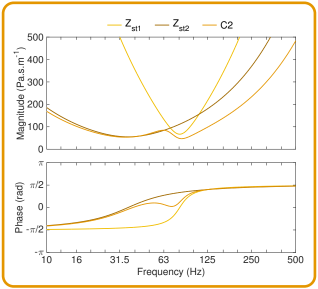


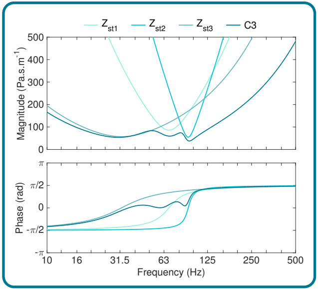
$Z_{st_{3-DOF}} = Z_{st_1} // Z_{st_2} // Z_{st_3}$ (C3)



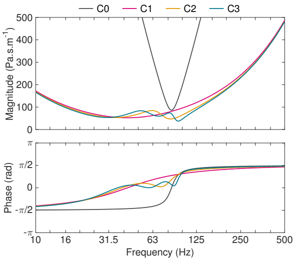
Performance analysis

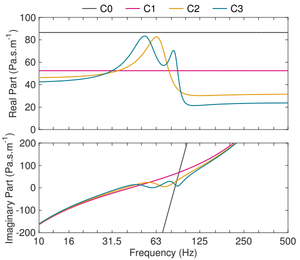
$Z_{st_{2-DOF}} = Z_{st_1} // Z_{st_2}$ (C2)





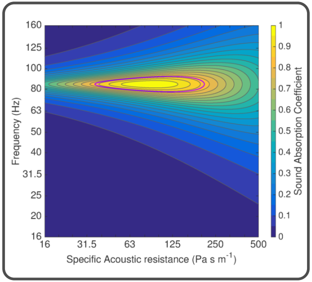
$Z_{st_{3-DOF}} = Z_{st_1} // Z_{st_2} // Z_{st_3}$ (C3)

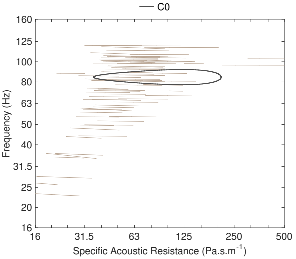




Modified Sound Absorption Coefficient

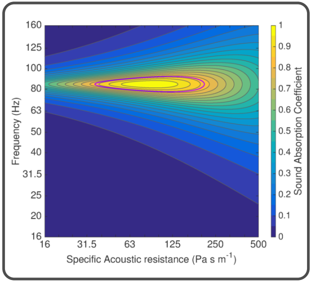
Basic config. (C0)

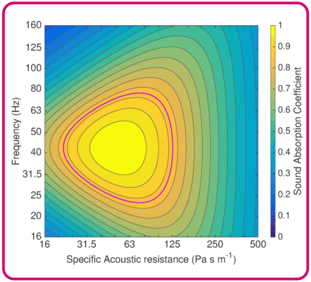




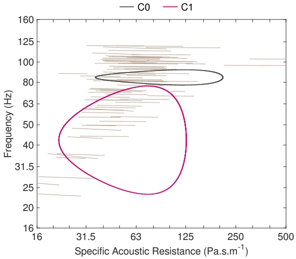
Modified Sound Absorption Coefficient

Basic config. (C0)



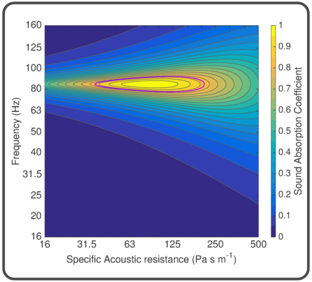


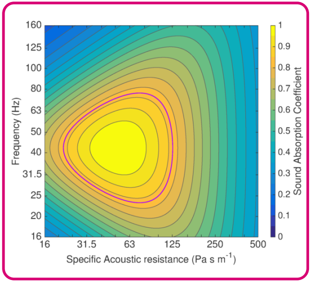
$Z_{st_{1-DOF}}$ (C1)



Modified Sound Absorption Coefficient

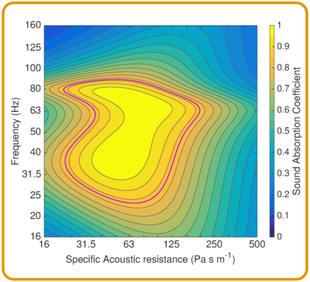
Basic config. (C0)

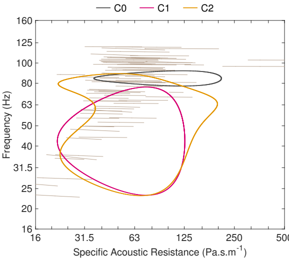




$Z_{st_{1-DOF}}$ (C1)

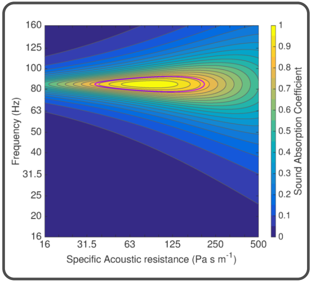
$Z_{st_{2-DOF}}$ (C2)



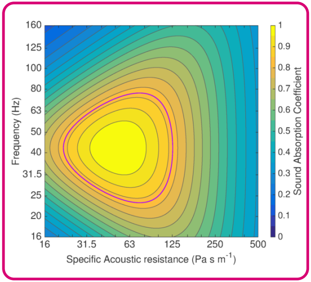


Modified Sound Absorption Coefficient

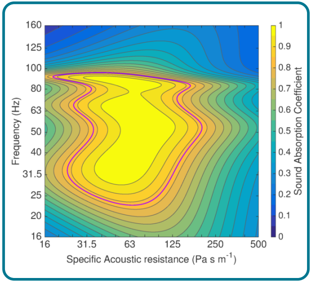
Basic config. (C0)

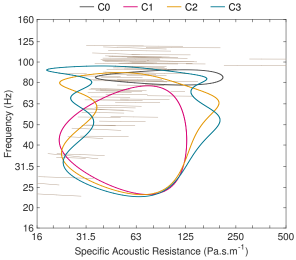


$Z_{st_{1-DOF}}$ (C1)



$Z_{st_{3-DOF}}$ (C3)



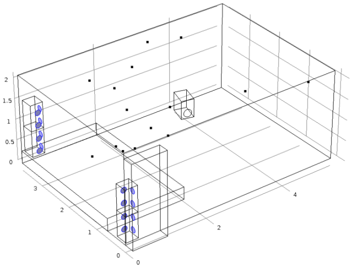


1 Electroacoustic absorbers design
- Electroacoustic absorbers: general principle
- Electroacoustic absorbers: hybrid sensor-/shunt-based control
- Performance assessment in 1D Case

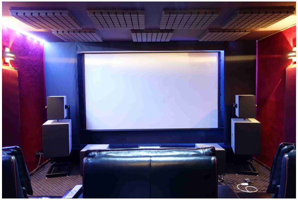
2 From 1D to 3D
- Identifying target Impedance in 3D
- Optimisation of Multiple Degree-of-Freedom Target Impedance

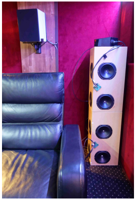
3 In situ performance Evaluation
- Frequency responses
- Modal decay times
- Spatial distribution of pressure

Experimental Setup: Small Room

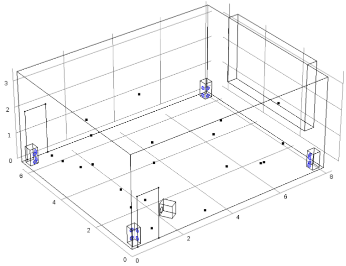


floor area $= 20.04\,\mathrm{m}^2$, volume $= 42.69\,\mathrm{m}^3$

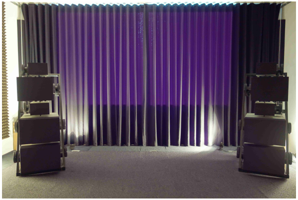


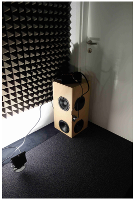


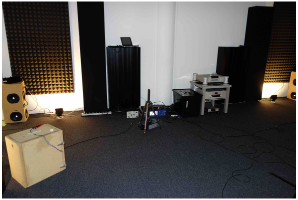
Experimental Setup: Medium Room



floor area $= 50.05\,\mathrm{m}^2$, volume $= 174.18\,\mathrm{m}^3$

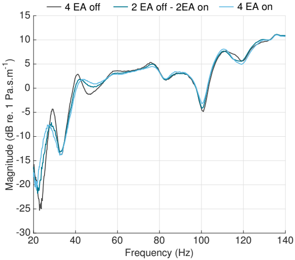




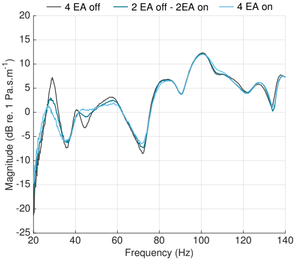


Modal Equalisation: Small Room

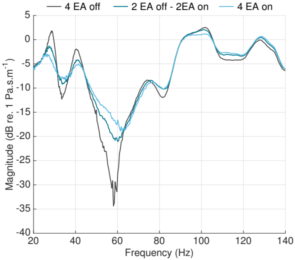
Mic #1



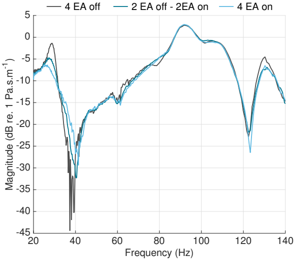
Mic #2



Mic #3

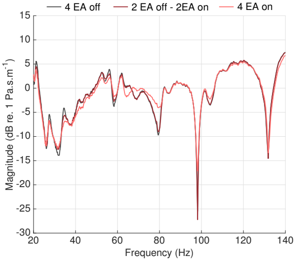


Mic #4

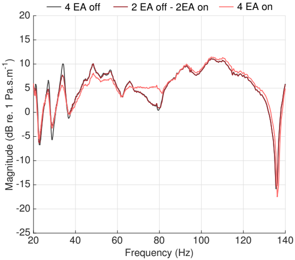


Modal Equalisation: Medium Room

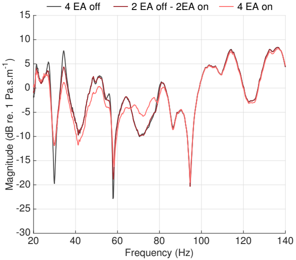
Mic #1



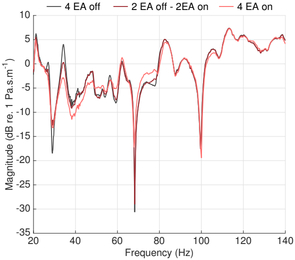
Mic #2



Mic #3

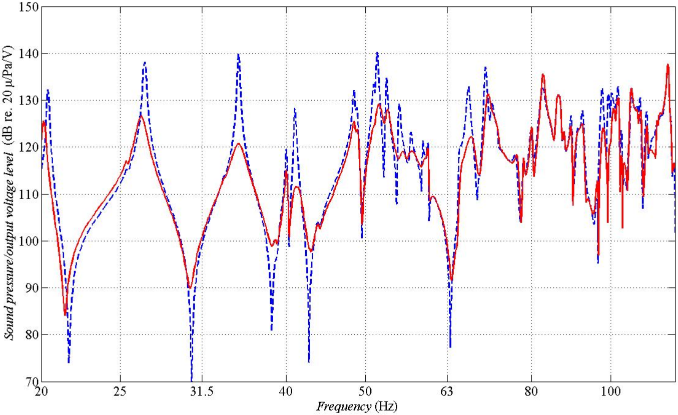


Mic #4



Evaluation of Modal Decay Times with RFP

Could be directly measured with sine excitations at resonance frequencies (provided they have been carefully identified)
Reverberation chamber:



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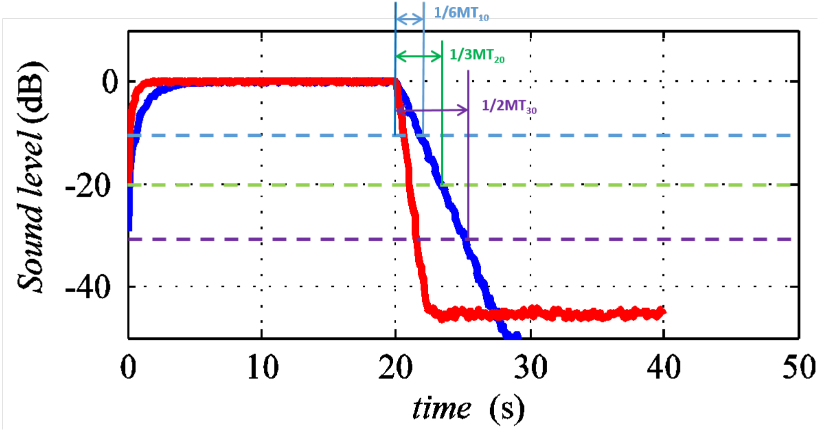
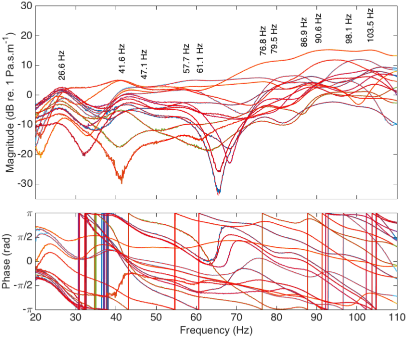


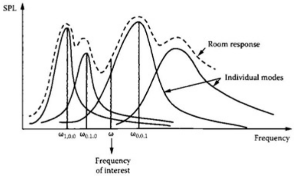
Evaluation of Modal Decay Times with RFP

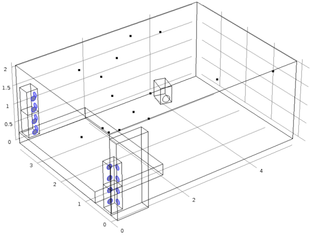
Modal decay time of mode n

$$MT_{60_n} = \frac{3\,ln(10)\,Q_n}{\pi\, f_{0_n}}$$

But in real rooms...



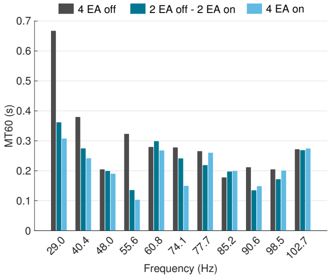


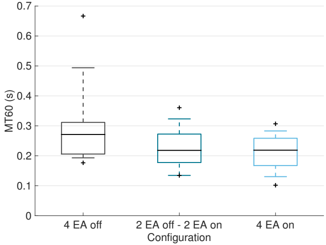


M. H. Richardson, D. L. Formenti, "Global curve fitting of frequency response measurements using the rational fraction polynomial method." *Proceedings of the Third International Modal Analysis Conference*, 1985.

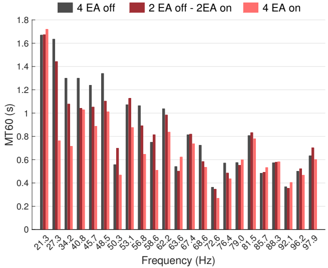
In situ evaluation of Modal Decay Times

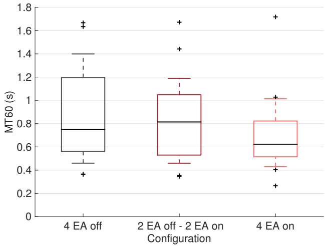
Small room





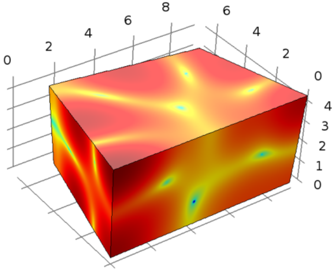
Medium room

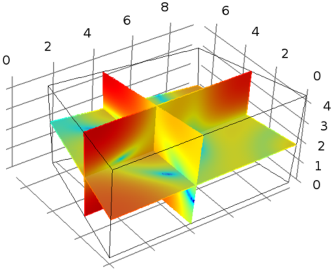




Is spatial sound pressure distribution accessible...?

- Target: Spatial evaluation of sound pressure field with and without absorbers
- Framework:
  - Input: sound pressure measurement at a limited number of microphone positions
  - Output: visualize/render sound pressure distribution within the whole room
- Challenges:
  - Limited number of actual microphones
  - If possible, limited computational requirements





… through Compressed Sensing?

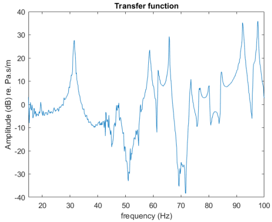
Compressed sensing[1]

- Reconstruction of a signals using far fewer samples than required by Nyquist theorem, given that the signal is sparse
- In acoustics: Reconstruction of the whole sound field in room using a much less number of measurements.
- Sparsity of sound field in room
  - Modal summation
  - Approximation of Modeshape function Ψ_n using plane waves summation

[1] Mignot et al., Low frequency interpolation of room impulse responses using compressed sensing, IEEE/ACM Trans. on Audio, Speech, and Language Processing, **22**(1) (2014)

Compressed Sensing framework: identifying the resonance frequencies

1. Measure a set of FRFs $H_m(\omega)$ (wrt a reference signal, eg. source velocity) at M microphone positions
2. Get the RIRs through inverse Fourier Transforms $h_m(t) = \mathscr{F}^{-1}\{H_m(\omega)\}$
3. Build a matrix of M vectors of size N_t
4. Define a database of N eigenvalues (ω_i, δ_i) within limit values
   Build a matrix (dictionary) of modes, such as $d_j(t) = e^{j\omega_i t}.e^{-\delta_i t}$
5. Project the measured signals on the dictionary
6. Select the column n_1 (corresponding to mode ω_{n1}, δ_{n1}) with the highest energy
7. Subtract each RIR $h_m(t)$ with the projected vector $a_{mn_1} e^{j\omega_{n_1} t}.e^{-\delta_{n_1} t}$
8. Repeat the operation until a set number of modes L (should be lower than M the number of microphones)
9. The outcome of this phase is the identification of L eigenfrequencies $(\omega_{n_i} + j\delta_{n_i})$



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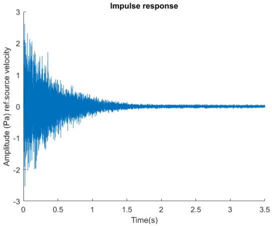

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$$\mathscr{M} = \begin{bmatrix} m_1(t_1) & m_1(t_2) & \dots & m_1(t_{N_t}) \\ m_2(t_1) & m_2(t_2) & \dots & m_2(t_{N_t}) \\ . & . & . & . \\ m_M(t_1) & m_M(t_2) & \dots & m_M(t_{N_t}) \end{bmatrix}$$

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$$\mathscr{D} = \begin{bmatrix} d_1(t_1) & d_2(t_1) & \dots & d_N(t_1) \\ d_1(t_1) & d_2(t_2) & \dots & d_N(t_2) \\ . & . & . & . \\ d_1(t_{N_t}) & d_2(t_{N_t}) & \dots & d_N(t_{N_t}) \end{bmatrix}$$

Compressed Sensing framework: identifying the resonance frequencies

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$$\mathscr{M} \cdot \mathscr{D} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ . & . & . & . \\ a_{M1} & a_{M2} & \cdots & a_{MN} \end{bmatrix}$$

Compressed Sensing framework: identifying the resonance frequencies

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$$\begin{bmatrix} a_{1n1} \\ a_{2n1} \\ . \\ a_{Mn_1} \end{bmatrix}$$

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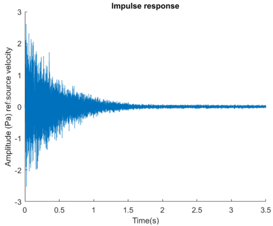

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$$\begin{bmatrix} \omega_{n_1} + j\delta_{n_1} \\ \omega_{n_2} + j\delta_{n_2} \\ . \\ \omega_{n_L} + j\delta_{n_L} \end{bmatrix}$$

Compressed Sensing framework: trying to recover the mode shapes

Making the assumption of plane wave decomposition:

$$p(t, \vec{X_m}) \approx \sum_{n_i, r \in \mathbb{N}^2} \alpha_{n_i,r} e^{-\vec{k_{n_i,r}} \cdot \vec{X_m}} e^{jk_{n_i} ct}$$

The second sparsity comes from the fact that each of the mode shapes can be approximated by a finite sum of plane waves coming from different direction (but with the same wave number $\forall r, ||\vec{k_{n_i,r}}|| = k_{n_i}$).

1. For each mode n_i, we build a set of r eigenvectors within a sphere of radius k_{n_i}
2. Projecting all M microphone (with known positions X_m) measurements on all r wavevectors $k_{n_i,r}$
3. LMS minimization allows retrieving the most likely wavevector $\vec{k_{n_i,r}}$

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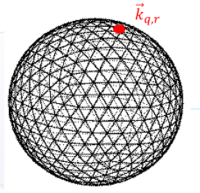


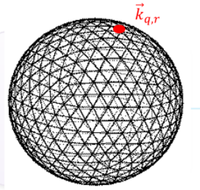
Compressed Sensing framework: trying to recover the mode shapes

Making the assumption of plane wave decomposition:

$$p(t, \vec{X_m}) \approx \sum_{n_i, r \in \mathbb{N}^2} \alpha_{n_i,r} e^{-\vec{k_{n_i,r}} \cdot \vec{X_m}} e^{jk_{n_i} ct}$$

The second sparsity comes from the fact that each of the mode shapes can be approximated by a finite sum of plane waves coming from different direction (but with the same wave number $\forall r, ||\vec{k_{n_i,r}}|| = k_{n_i}$).

1. For each mode n_i, we build a set of r eigenvectors within a sphere of radius k_{n_i}
2. Projecting all M microphone (with known positions X_m) measurements on all r wavevectors $k_{n_i,r}$
3. LMS minimization allows retrieving the most likely wavevector $\vec{k_{n_i,r}}$



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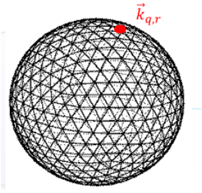
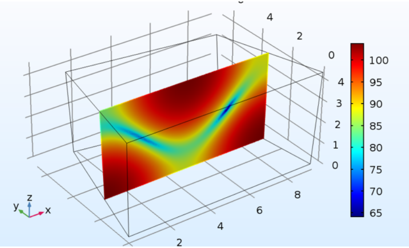


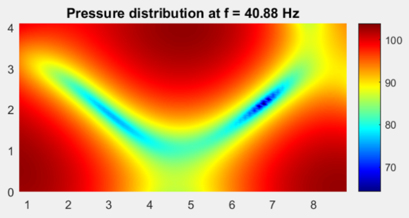
Compressed Sensing reconstruction: results so far

- Goal: sound field reconstruction (in thereverberation chamber)
- Setup: 50 randomly placed microphones measurements in the room
- For now: COMSOL simulations (as ground truth) and validation of compressed Sensing interpolation out of a limited number of microphones

FEM modeling (40.88 Hz)



CS result (40.88 Hz)



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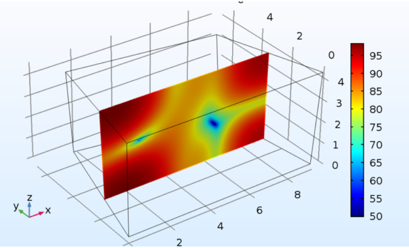
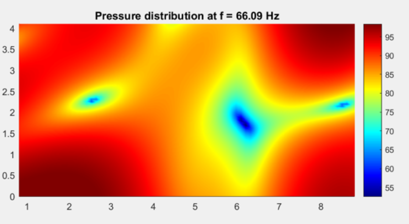
FEM modeling (66.09 Hz)



CS result (66.09 Hz)



Summary of results

- Design guidelines for setting electroacoustic absorbers
  - Impedance control through multiple degree-of-freedom resonators
  - Hybrid sensor-/shunt-based impedance control
- Performance optimisation
  - Definition of new performance metrics
  - Methodology for performance improvement
- Performance evaluation for modal equalisation
  - Development of demonstration prototypes
  - Objective evaluation in real rooms
  - Subjective evaluation of music playback in rooms (not presented here)

Perspectives

- Absorber design
  - Ratio surface/mass
  - Active panels controlled by electrodynamic inertial actuators
  - Development of electroacoustic absorption-diffusion systems
- Performance optimisation
  - Target impedance (reactive terms in simulations)
  - Calibration of electroacoustic absorbers directly in the room
- Performance evaluation
  - Spatial sound field reconstruction (compressive sensing)

Thank you for your Attention

Thanks to: Etienne Rivet, Sami Karkar, Romain Boulandet, and Thach Pham Vu

Most of the presented results are taken from
Etienne Rivet PhD Thesis No. 7166 (open access)

Open access paper:
E. Rivet et al, Acta Acustica, **103**(6), pp. 1025-1036 (2017)